



Measuring the direction of gravity acceleration

Kamil Madáč^{a,*}, Andrej Madáč^b, Kamil Madáč jr.^c, Peter Popovec^d

^a Faculty of Mechanical Engineering, Technical University of Kosice, Letná 9, 042 00 Kosice, Slovak Republic

^b Retiree, Hroncova 15, 040 01 Kosice, Slovak Republic

^c Entrepreneur, Internet Technologies and icloud programator, Slovak Republic

^d Faculty of Electrical Engineering and Informatics, Technical University of Kosice, Slovak Republic

ARTICLE INFO

Keywords:

Magnitude of gravity
Direction of gravity
Atmosphere
Weather
Moon

ABSTRACT

The article contains a description of a measuring device, which is primarily designed to record changes in the direction of gravitational acceleration on the Earth's surface. Secondly, the measuring device records the inclination of the Earth's crust. Analysis of the measured data showed that the direction of gravitational acceleration on the Earth's surface is affected by two objects. The first object is the atmosphere of the Earth (weather). Changes in the distribution of atmospheric density due to the Earth's motion cause changes in the direction of gravitational acceleration on the Earth's surface. The second object is the Moon. The Moon's gravitational field creates a "gravitational shadow" in the Earth's gravitational field. During the Full Moon, the gravitational shadow reaches the surface of the Earth.

1. Introduction

According to current knowledge, the mass of the celestial body curves space-time. It also applies to our Earth. At every point on the Earth's surface, the curvature of space-time has its magnitude and direction. Many scientific teams are involved in measuring the parameters of the Earth's gravitational field. In addition to measurements on the Earth's surface, sophisticated measurements from the Earth's orbit are also performed.

One of the best-known measurements of gravity on the Earth's surface is the LIGO experiment. This experiment was designed to record a gravitational wave. In 2015, it succeeded.

The satellites LAGEOS 1 (1976) and LAGEOS 2 (1992) 's main goal was to measure the Earth's shape - geoid. Earth's geoid measurements make up a large part of not only satellite measurements but also terrestrial measurements [1,2,3,4,5].

The satellites Probe A (1977) and Probe B 2006 were designed on measuring the hypothetical geodetic effect of space-time curvature (warping) around material bodies as well as space-dragging (or Lens-Thirring phenomenon) by rotating objects. Experiments were not successful.

Gravimetric methods are used to measure the magnitude of the gravitational acceleration on the Earth's surface. Gravimetry is used to determine inhomogeneities in the earth's crust. Searching the cavities,

solving problems of oil geophysics, hydrogeology, in environmental and deposit geology, but also in solving many other problems [6,7,8,9,10,11].

Another group is measurements to define the global gravitational field. Since the GOCE satellite launch, the global gravitational field has been available in a resolution that presents challenges in studying tectonic and geological features. Gravity models are publicly available, as is the necessary software to calculate the fields [12,13].

Experiments refining the measurement of gravity parameters, or designed for special solutions using gravity [14,15,16,17,18,19].

During a visit to CERN in 2019, we recorded information about testing a measuring device to monitor the Earth's crust's inclination. Unlike traditional seismometers, the Precision Laser Inclinator (PLI) measures the effect of the inclination of the earth's crust on the liquid surface. The measurement is performed by directing the laser light at the liquid and monitoring the light reflection. The aim is to predict earthquakes when performing measurements at the interface of continental plates.

So far, we have not recorded any contributions aimed at measuring the direction of gravity on the Earth's surface.

2. Experimental methods

A well-known sensor of the direction of gravity on the Earth's surface

* Corresponding author.

E-mail address: kamil.madac@tuke.sk (K. Madáč).

<https://doi.org/10.1016/j.measurement.2021.109908>

Received 6 February 2021; Received in revised form 11 July 2021; Accepted 18 July 2021

Available online 24 July 2021

0263-2241/© 2021 Elsevier Ltd. All rights reserved.

is the classical plumb line used in construction. Instead of a plumb line suspended at one point, we used the pendulum suspended at two points. It allows measuring the direction of gravity in the plane.

Monitoring the change of the direction of gravity in the meridian plane (North-South) is ensured by the points of the hinge lying in the direction of the parallel, Fig. 1. Monitoring of the change of the direction of gravity in the plane of the parallel (East-West) is ensured by the points of the suspension lying in the direction of the meridian.

Inductive sensors performed initial measurements in 2002. These sensors have sufficient sensitivity but have two disadvantages. The first disadvantage is the measurement output - voltage. The second disadvantage is the everyday need to balance the measuring circuit. It means that this sensor did not allow continuous (uninterrupted) recording of data. After consulting with Renishaw employees (2012), we replaced the inductive sensor with a TONIC laser incremental encoder [20], Fig. 2.

The gold-plated RGSZ20-S scale enables measurements with the sensitivity of one nanometer. The sensitivity of the measurement depends on the sensor used. The TONIC 1000-10A sensor enables measurements with the sensitivity of 100 nm.

TONIC general specifications

Power supply $5\text{ V} \pm 10\%$

Temperature (system) Storage $-20\text{ }^{\circ}\text{C}$ to $+70\text{ }^{\circ}\text{C}$ Operating $0\text{ }^{\circ}\text{C}$ to $+70\text{ }^{\circ}\text{C}$

Mass Readhead 10 g, Interface 100 g

Readhead cable (standard) Double shielded, outside diameter $4.25 \pm 0.25\text{ mm}$

Scale technical specifications

Scale type RGSZ20-S: High stability steel tape, gold plated with polymer protection, and self-adhesive backing tape.

Scale period $20\text{ }\mu\text{m}$

Linearity RGSZ20-S: $3\text{ }\mu\text{m/m}$ RGSZ20-P: $\pm 5\text{ }\mu\text{m/m}$

Form $0.2\text{ mm} \times 6\text{ mm}$ ($H \times W$) includes adhesive

Installation temperature $15\text{ }^{\circ}\text{C}$ to $35\text{ }^{\circ}\text{C}$

Due to the use of an incremental laser encoder, the uncertainty of the accuracy of the RGSZ 20S scale and the operation of the quadrature decoder corresponds. Temperature shift compensation is eliminated by stabilizing the measuring space temperature at $19\text{ }^{\circ}\text{C}$. Due to the measurement interval (days, months, and years), the time shift is unnecessary.

In Fig. 3 there is a photograph of the supporting frame of a measuring device with two pendulums following the direction of gravitational acceleration. The closed supporting frame is made of a 3 mm stainless steel sheet. The supporting frame is not affected by a change in external forces during the measurement. We assume that a closed support frame does

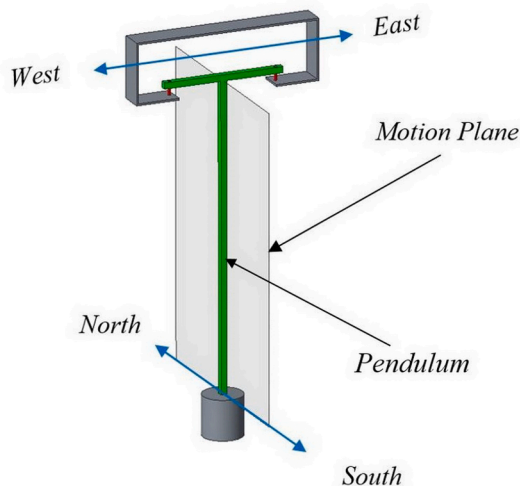


Fig. 1. Scheme of measuring the direction of gravity. Measuring the position of the pendulum in the North-South plane.

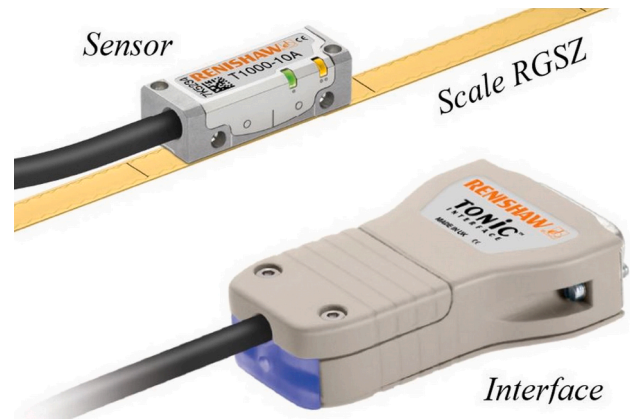


Fig. 2. Laser incremental sensor T1000-10A, interface TONIC, and Scale RGSZ20-S.

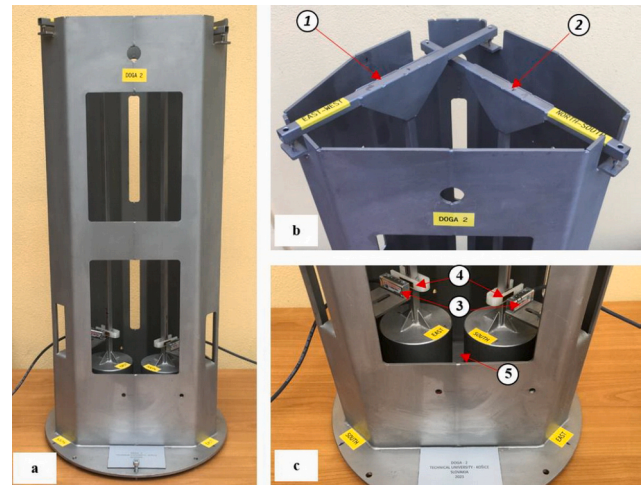


Fig. 3. Mechanics of measuring equipment. a – Measuring device with pendulums, b – Detail of pendulums suspension, c – Detail of submerged pendulums, 3 - Laser sensors T-1000-10A, 4 - Scales RGSZ20 -S, 5 - Bottom of the bathtub.

not affect the measurement accuracy. In Fig. 4 there is a photograph of electronics of measuring equipment. electronics of measuring equipment see Fig. 4.

Fig. 5 is a block diagram of a measuring chain. At a distance of 500 mm from the pendulum suspension, a gold-plated RGSZ20-S scale is

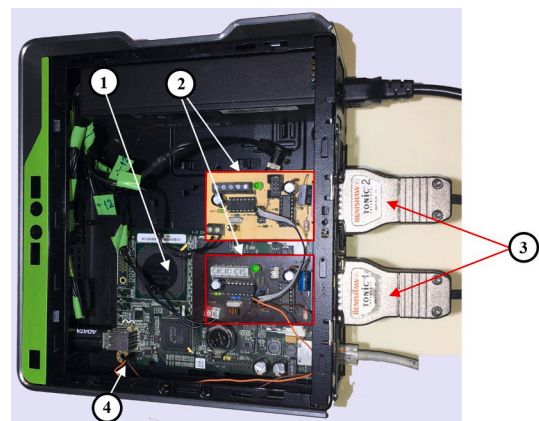


Fig. 4. Electronics of measuring equipment. 1 - Source, 2 - ALIX system board, 3 - Quadrature Encoders, 4 - TONIC interfaces.

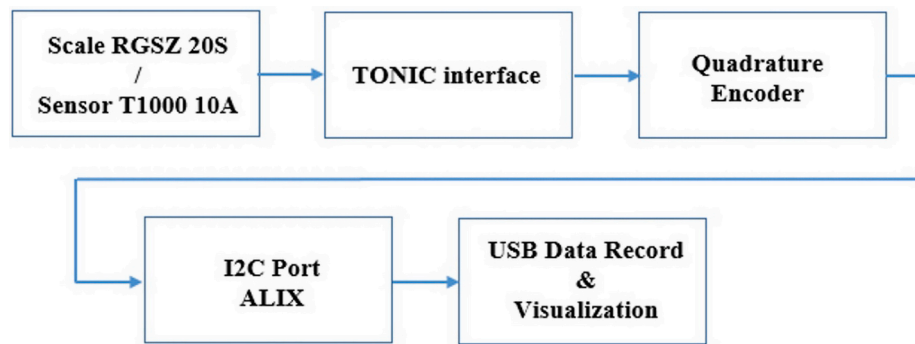


Fig. 5. Block diagram of the measuring device.

placed, on which vertical grids are made. An incremental laser sensor records the change in the position of the pendulum.

The weight (pendulum) position is measured at a distance of 500 mm from the hinge. The frequency of reading the pendulum positions is 10 Hz. Data recorded during one hour is stored in a separate file. Hourly datasets from 2018, 2019, and 2020 are available online [22].

In 2015, the upgrade of the measuring equipment was completed. The measurement device is placed on a concrete slab in a thermally insulated pit, Fig. 6. A three-meter deep pit is located outside the built-up area. The temperature in the pit is stabilized at 19 °C.

The motion of the pendulums can be monitored online at <http://kyvadlo.sjf.tuke.sk/KYVADLO-local/kyvadlo.html>

3. Results

3.1. The inclination of the Earth axis

The Earth's surface is essentially the bottom of the "ocean of air." The mass of the atmosphere (ocean of air) is less than a millionth of the Earth's mass. The mass of the Earth's atmosphere has a negligible effect on gravity's magnitude on the Earth's surface. However, this does not apply to the influence of the atmosphere on the direction of gravity.

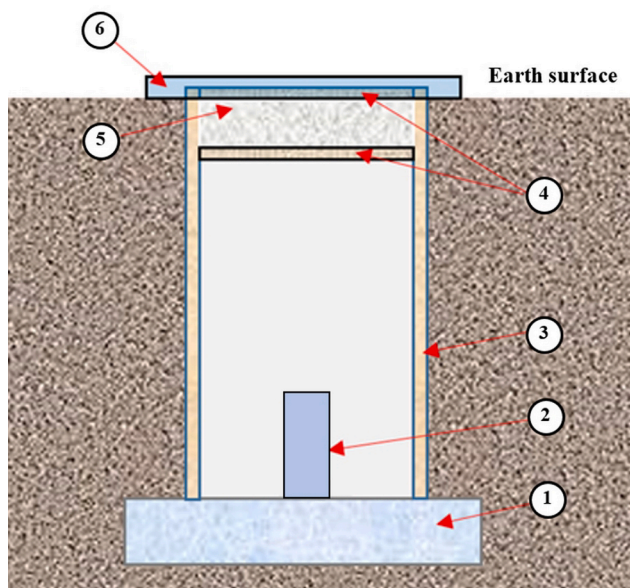


Fig. 6. Location of the measuring device three meters below the ground. 1 - Concrete foundation, 2 - Measuring device, 3 - Round concrete parts, 4 - Polystyrene insulation (10 cm), 5 - Cotton wool insulation (50 cm), 6 - Cover Pendulum North-South: http://kyvadlo.sjf.tuke.sk/LOG/kyvadlo_i2c-0_0x60/ Pendulum East-West: http://kyvadlo.sjf.tuke.sk/LOG/kyvadlo_i2c-0_0x61/. The geographical position of Measurement: 48°38'11 North, 21°17'57 East.

The air density depends on pressure, temperature, and humidity. Global weather is affected by the inclination of the Earth's axis of rotation during its orbit around the Sun. In winter, less solar energy falls in the northern hemisphere than in summer. It corresponds to the position of the pendulum during the Earth's orbit around the Sun.

The record of the pendulum motion during 2018 (North-South) is shown in Fig. 7. The amplitude of the pendulum motion reached 70 μm (28 arcsec). The direction of gravity reaches extreme values in spring and autumn.

The record of the pendulum motion during 2019 (North-South) is shown in Fig. 8. The annual amplitude of the pendulum position reached 27 μm (11 arcsec).

The record of the pendulum motion during 2020 (North-South) is shown in Fig. 9. The annual amplitude of the pendulum position reached 70 μm (28 arcsec).

From annual periods of pendulum motion in the north-south direction, it is clear that extreme values (spring and autumn) are related to the inclination of the Earth's axis of rotation to the Sun. The annual periods of pendulum motion express the effect of global weather on gravity's direction at the measuring point.

3.2. The Earth rotation

Daily periods are visible in the record of annual periods. The first daily periods appear after the vernal equinox. There is an increase in the difference between day and night temperatures at the measuring point (weather). Cloudless weather means high pressure with a constant distribution of atmospheric density. The pendulums reach extreme values in the morning and the afternoon.

Fig. 11 is the course of daily temperatures on the same day. A comparison of Figs. 10 and 11 shows that the extreme values correlate with each other.

Cloudy weather means low pressure with variable distribution of atmospheric density. The course of pendulums motions during 24 h is non-periodic, Fig. 12.

Fig. 13 is the course of daily temperatures on the same day. A comparison of Figs. 12 and 13 shows that the waveforms do not correlate with each other.

At the end of October (after the autumn equinox), the difference between day and night temperatures decreases. Finally, the daily periods disappear completely, Fig. 14. It is repeated every year, but not on the same day.

3.3. The Moon

The Moon orbits in the Earth's gravitational field. It means that the Moon creates a gravitational shadow in the Earth's gravitational field, Fig. 15. The image results from a simulation using the ANSYS CFX - computational fluid dynamics software [21]. The force lines of gravity of the Moon's gravitational field create a gravitational shadow in the

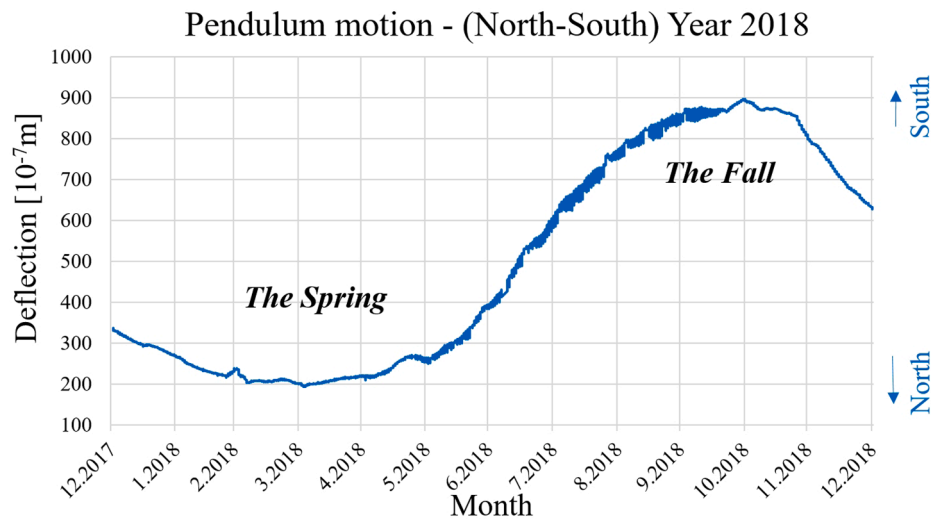


Fig. 7. Annual period of pendulum motion in the North-South direction (2018).

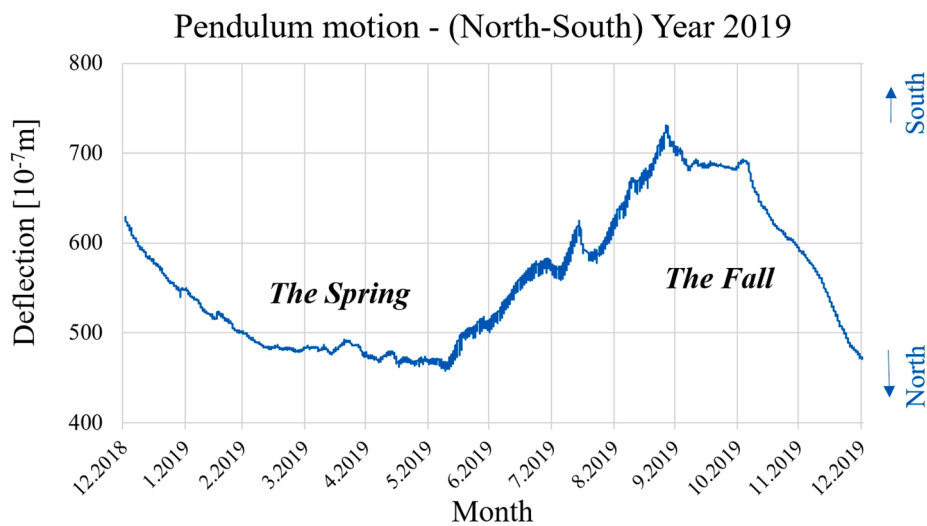


Fig. 8. Annual period of pendulum motion in the North-South direction (2019). Note: The course of pendulum positions during the summer is significantly influenced by global weather.

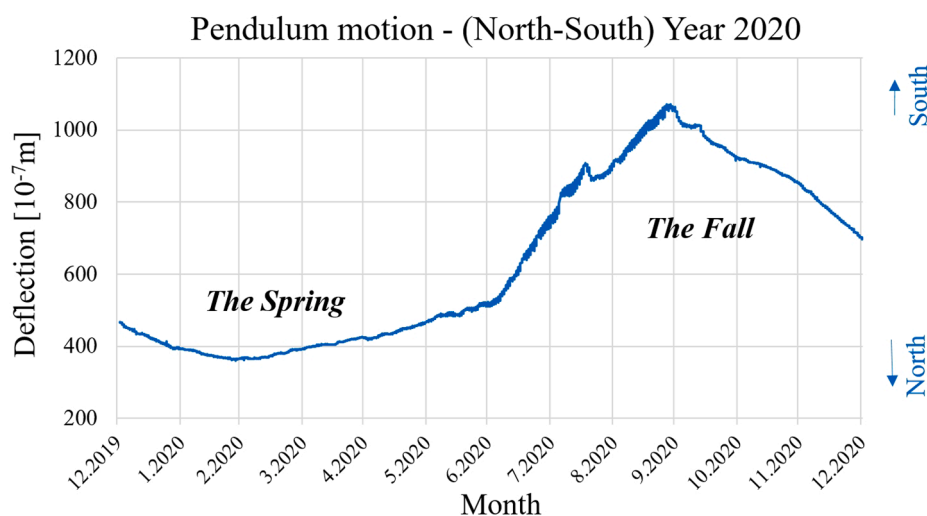


Fig. 9. Annual period of pendulum motion in the North-South direction (2020).

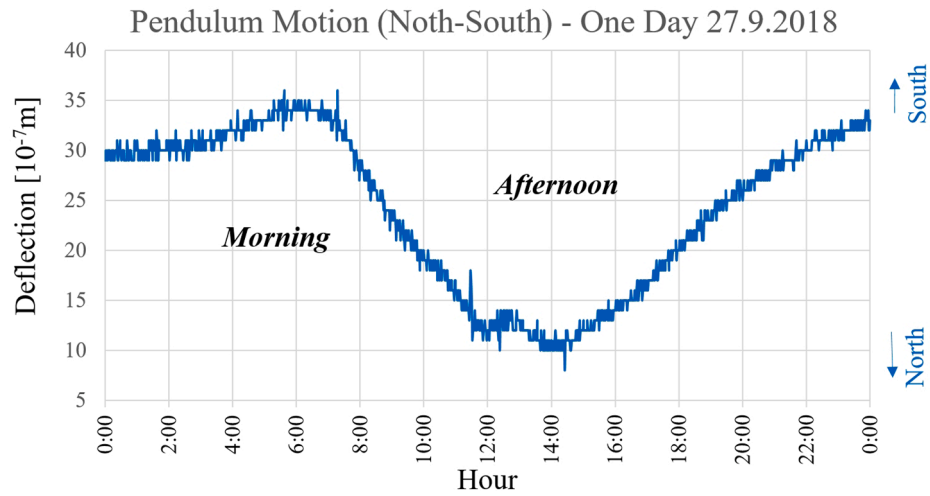


Fig. 10. Daily periodical motions of pendulums (September 27, 2018).

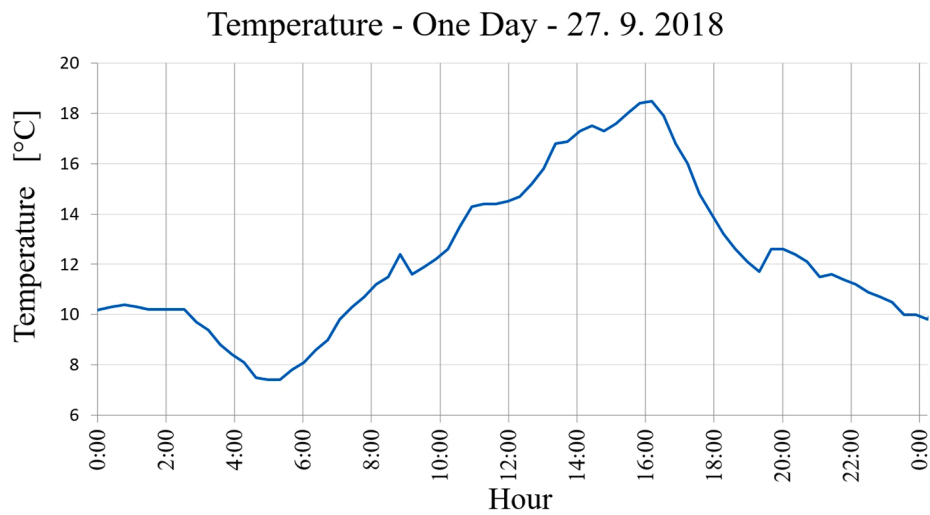


Fig. 11. Daily temperature (September 27, 2018).

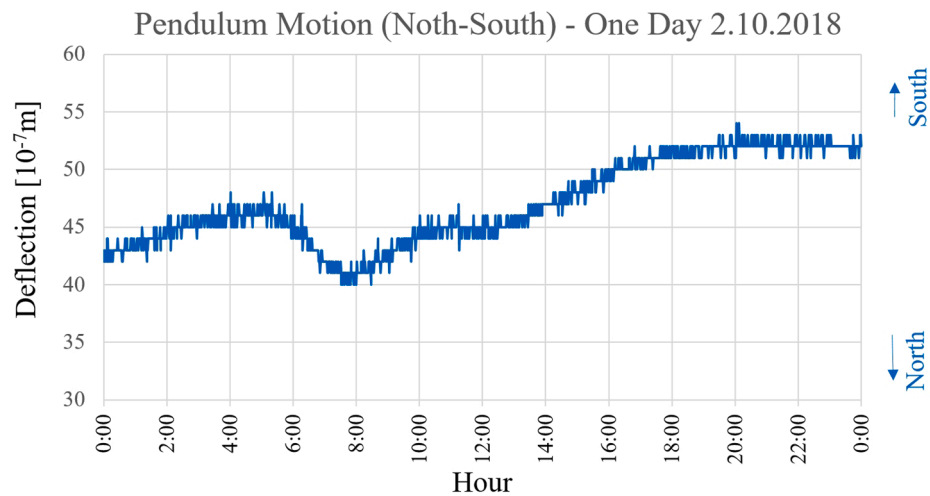


Fig. 12. Non-periodic motions of pendulums in case of cloudy weather (October 2, 2018).

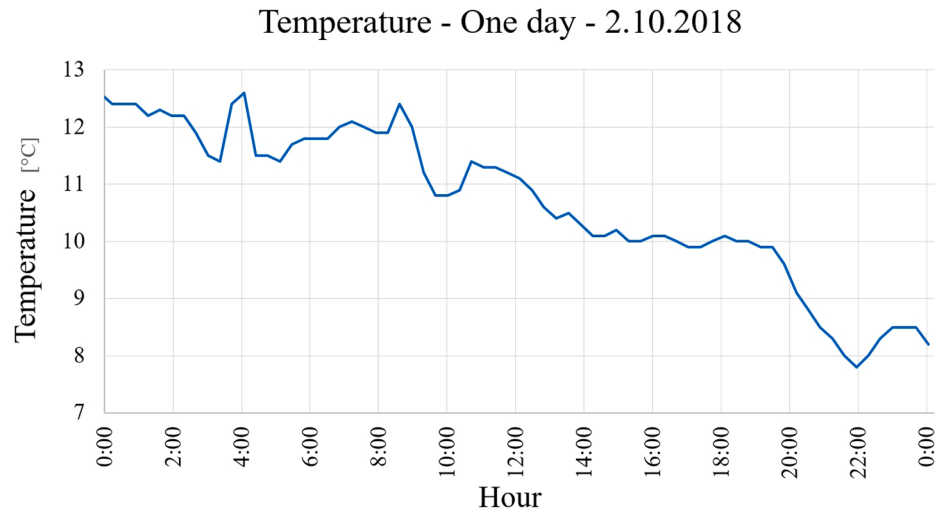


Fig. 13. Daily temperature (October 2, 2018). Note: The direction of gravitational acceleration at the measurement site is affected by the distribution of the atmosphere's density around the measurement. This distribution may not correspond to the temperature profile at the measuring point.

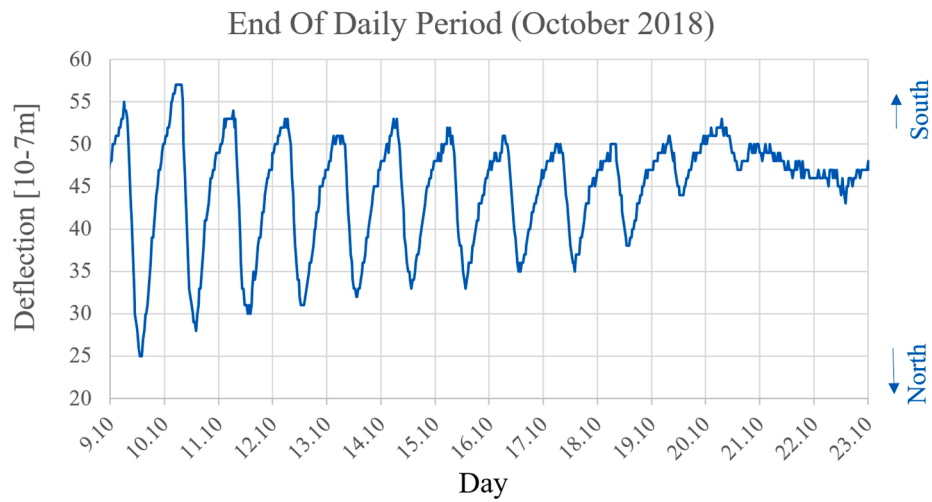


Fig. 14. Termination of daily periods in October of 2018.

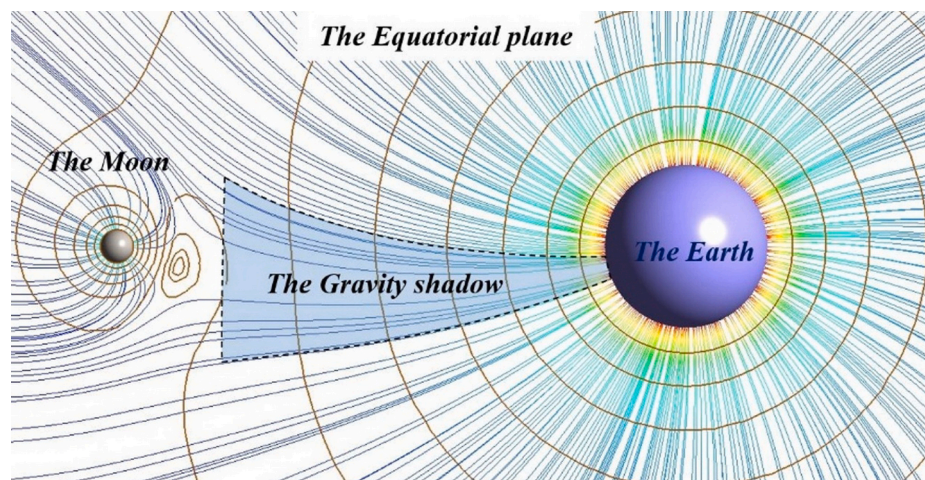


Fig. 15. The gravitational shadow of the Moon in the rotating gravitational field of the Earth. In addition to the force lines, the contours of the gravitational fields are also displayed.

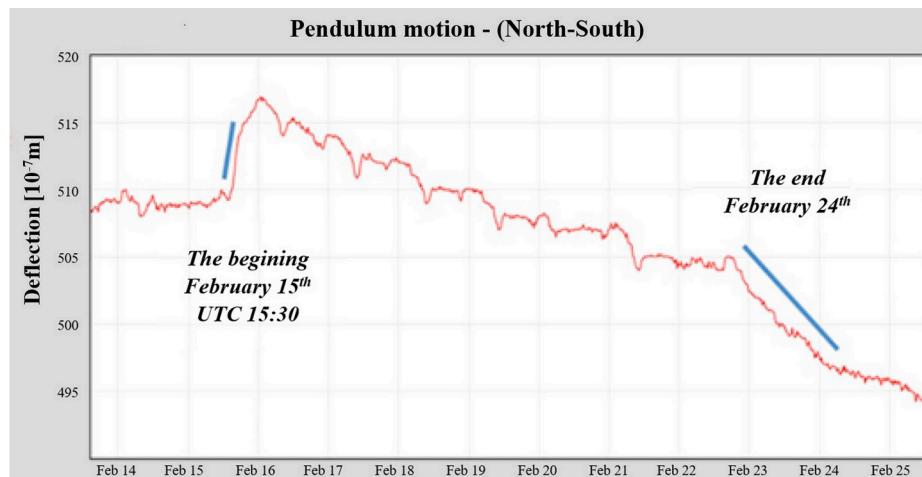


Fig. 16. Influence of the Moon on the motion of the pendulum (North-South).

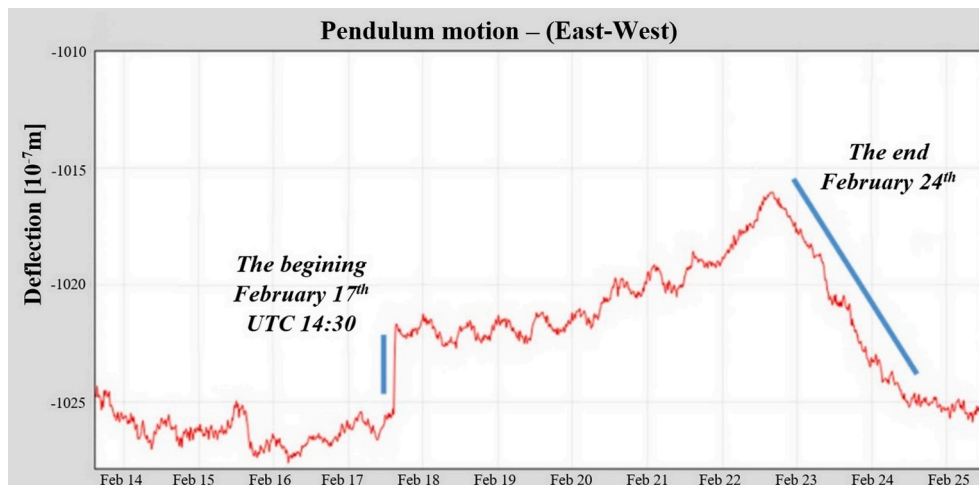


Fig. 17. Influence of the Moon on the motion of the pendulum (East-West).

Earth's gravitational field.

The gravitational shadow on the Earth's surface has not been recorded for long seventeen years. The cause is a position of measuring point - too far from the equator.

Figs. 16 and 17 show the influence of the gravitational shadow of the

Moon on the motion of pendulums (Fool Moon - February 19). Because the pictures illustrate the effect of the Moon's gravity, we used a presentation of this effect from a website.

The following facts follow from the pictures.

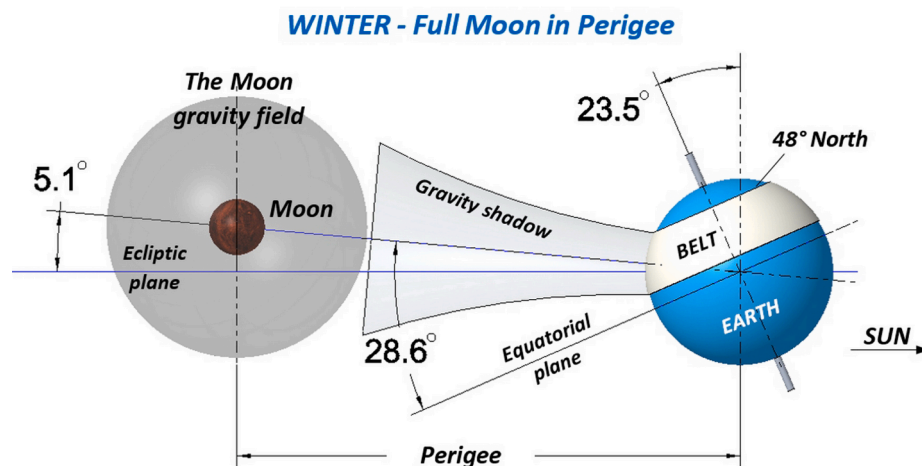


Fig. 18. The belt of the Moon's gravitational shadow on the Earth's surface in winter. The Moon is in Apogee.

- The influence of the gravitational shadow of the Moon began at the time (15:30 UTC) when the Moon was below the horizon in the northeast direction.
- The Moon's gravitational shadow on the second pendulum in the east–west direction until February 17 at 14:30 UTC, i.e., 47 h later.
- The gravitational shadow of the Moon acted uninterrupted on both pendulums until February 24.

The explanation of the above facts allows the assumption of the rotation of the Earth's gravitational field. The speed of rotation of the gravitational field is many times higher than the speed of rotation of the Earth.

The gravitational shadow of the Moon orbits the surface of the Earth many times an hour. The gravitational shadow of the Moon forms a belt on the Earth's surface. The belt moves in the direction of the meridian, Figs. 18 and 19.

When the edge of the belt hits the measuring point does not depend on the relative position of the Moon and the measuring point, but only on the belt's position relative to the equator!

The gravitational shadow of the Moon begins to act on the gravitational acceleration in the east–west direction at the moment when the orbital motion of the Moon “brakes” the gravitational field of the Earth. The speed of rotation of the Earth's gravitational field in the Moon's orbit is higher than the Moon's speed. It is reflected in the pendulum's movement in the east–west direction when the gravitational shadow (not the belt) begins to affect the direction of the gravitational acceleration in the east–west direction.

The belt of the gravitational shadow of the Moon acts at the measuring point continuously for several days. The cause is the belt width and relatively slow movement of the belt in the meridian direction.

The speed of movement of the belt in the direction of the meridians corresponds to the orbital period of the Moon around the Earth. The speed of motion of the belt is approximately $4 \times (23.5^\circ + 5.15^\circ) = 114.6^\circ$ in 27 days.

During the full Moon (February 2019), three conditions were fulfilled to record the Moon's gravitational shadow's influence at the measurement site, 48°N .

1. The inclination of the Earth's axis of rotation gravitationally (not optically) with respect to the Sun was close to the maximum.
2. The Moon was in its orbit near the maximum above the plane of the ecliptic.
3. The width of the belt was maximum because the Moon was close to Perigee.

The first condition expresses the fact that the Sun's gravitational field rotates. As we know, optically, the inclination of the Earth's axis with respect to the Sun is maximum on December 21. The record of the gravitational shadow of the Moon means that the gravity of the Earth's axis relative to the Sun is maximal during February.

The first and second conditions (together) are subject to the so-called **Metonic cycle**. Every 19 years, the same phase of the Moon occurs in the same place as distant stars.

The third condition expresses that the compliance of all three conditions at 48° north latitude is not subject to the “**Metonic Cycle**.” We were lucky to record the influence of the Moon in 2019. Before that, all three conditions were met during the full Moon in February 1905.

3.4. The earthquake

Of course, the motion of the Earth's crust also affects the motion of the pendulums. Fig. 20 shows an example of the impact of the earthquake in Chile, 2015. Because the pictures illustrate the effect of earthquake, we used a presentation of this effect from a website

On the left is the course in 24 h, on the right in 10 min. Earthquake distance from measuring point 12 656 km.

The amplitude of the pendulum deflections reached $10\ \mu\text{m}$. The frequency of the Earth's crust was approximately $1/20\ \text{Hz}$.

4. Discussion and conclusion

Analysis of the measured data proves that the Earth's motion (change of weather) causes periodic changes in gravity's direction on the Earth's surface. It should be emphasized that the measured changes in the direction of gravity are relative.

The inclination of the Earth's axis. The mass density distribution of the atmosphere (weather) around the measurement point depends on the Earth's axis of rotation to the Sun. Global weather has a decisive influence on the annual periods of the pendulum motion.

Earth's rotation. The motion of the pendulums for 24 h is affected by local weather. There are considerable differences in day and night temperatures during the Earth's rotation from May to October. In the case of cloudless weather, there are daily periods of changes in the direction of gravity. After the autumn equinox, the amplitudes of the daily periods attenuate. Eventually, the daily periods will disappear altogether.

The Moon. The center of the Moon's gravitational shadow moves within $\pm 28.5^\circ$ of the plane of the ecliptic during the year. The Earth's surface will reach the gravitational shadow only in the full Moon. In 2019, the gravitational shadow of the Moon changed the direction of the

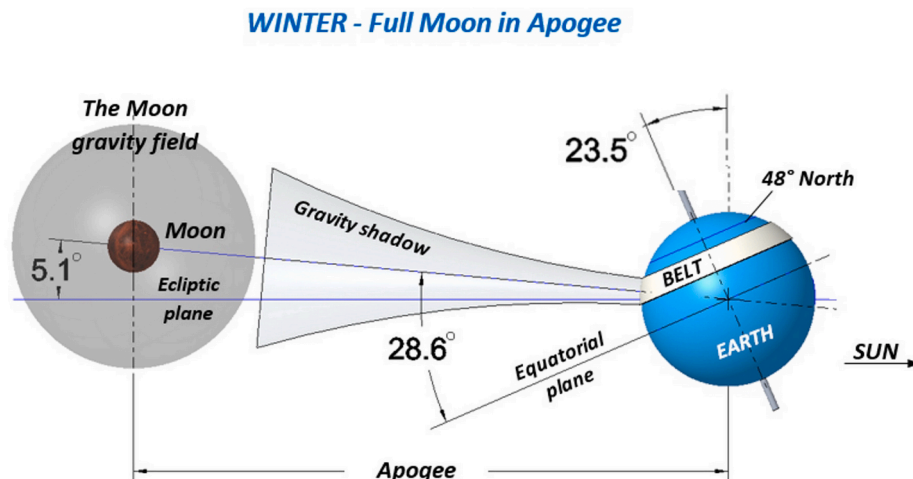


Fig. 19. The belt of the Moon's gravitational shadow on the Earth's surface in winter. The Moon is in Perigee.

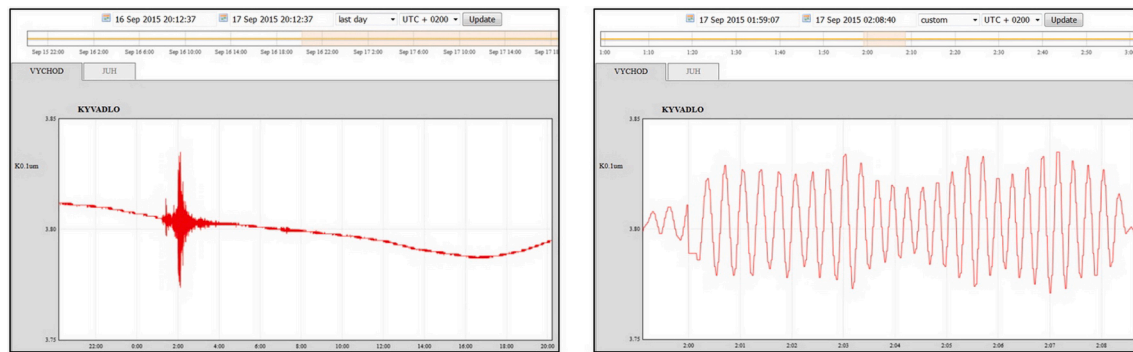


Fig. 20. The motion of the pendulum during the earthquake in Chile - July 9, 2015. **Magnitude: 8.3°**, **Place 31.573°S 71.674°W**, **Time: 2015 – 09–17 00:54:33 UTC + 02.**

gravitational acceleration on 48° north latitude for nine days, from 15 to February 24. The width of the gravity shadow band reached a value of approximately 40°. A measuring device located on Earth's tropics should record a gravitational shadow's effect several times a year.

Our University in Košice, in cooperation with the Vatican Observatory [22], is preparing the implementation of the DOGA2 measuring device at the observatory in Arizona-USA. Scientists from the observatory will study the effect of the inclination of the Earth's crust on the positioning of the telescope - Pointing Model [23]. Our measurements will focus on the record of the Moon's influence on the direction of gravitational acceleration. We assume that the position of the measuring device at 32° North latitude will make it possible to record the effect of the Moon's gravity at least once a year.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgment

We want to thank the Faculty of Mechanical Engineering of the Technical University in Košice for providing the experiment's implementation background. We want to thank my son *Kamil* and colleagues from the Technical University of Košice (*Ján Král jr.*, *Peter Popovec*, *Jaroslav Melko*, *Gabriel Fedorko*). Thanks also go to Milan Guba for his help in the production of the supporting frame and pendulums.

References

- [1] I.N. Tziavos, G.S. Vergos, S.P. Mertikas, A. Daskalakis, V.N. Grigoriadis, A. Tripolitis, The contribution of local gravimetric geoid models to the calibration of satellite altimetry data and an outlook of the latest GOCE GGM performance in Gavdos, *Adv. Sp. Res.* 51 (8) (2013) 1502–1522, <https://doi.org/10.1016/j.asr.2012.06.013>.
- [2] V. Corchete, The high-resolution gravimetric geoid of Italy: ITG2009, *J. African Earth Sci.* 58 (3) (2010) 580–584, <https://doi.org/10.1016/j.jafrearsci.2010.05.010>.
- [3] S. Doganalp, H.Z. Selvi, Local geoid determination in strip area projects by using polynomials, least-squares collocation and radial basis functions, *Meas. J. Int. Meas. Confed.* 73 (2015) 429–438, <https://doi.org/10.1016/j.measurement.2015.05.030>.
- [4] B.D. Tapley, S. Bettadpur, J.C. Ries, P.F. Thompson, M.M. Watkins, GRACE measurements of mass variability in the Earth system, *Science* (80-.). 305 (2004) 503–505. <http://doi.org/10.1126/science.1099192>.
- [5] A. Kenyeres, M. Sacher, J. Ihde, H. Denker, U. Marti, EUVN_DA: Establishment of a European continental GPS/leveling network, in: 1st Internat. Symp. Internat. Gravity F. Serv., IAG Symposia, Springer Verlag, Istanbul, 2006: pp. 1–14.
- [6] K. Medved, M. Kuhar, B. Koler, Regional gravimetric survey of central Slovenia, *Meas. J. Int. Meas. Confed.* 136 (2019) 395–404, <https://doi.org/10.1016/j.measurement.2018.12.065>.
- [7] L. Xing, L. Bai, X. Niu, P. Sang, A new and high-precision gravity base network in the south of the Tibetan Plateau, *Geod. Geodyn.* 11 (4) (2020) 258–264, <https://doi.org/10.1016/j.j.geog.2020.05.001>.
- [8] A. Humeniuk, O. Bezvesilna, M. Bogdanovskiy, V. Yanchuk, S. Semerikov, S. Chukharev, S. Sakhno, A. Striuk, V. Osadchyi, V. Solovieva, T. Vakaliuk, P. Nepochurenko, O. Bondarenko, H. Danylichuk, Information and measurement system for determining the acceleration of gravity based on a ballistic gravimeter with a twodimensional video system, *E3S Web Conf.* 166 (2020) 05006, <https://doi.org/10.1051/e3sconf/202016605006>.
- [9] F.E. Kemgang Ghoms, N. Ribeiro-Filho, R. Baldez, R. Tenzer, C.M. Martins, C. Chisenga, S. Nguia, R. Nouayou, Identification of Cameroon's geological structures through a gravity separation and using seismic crustal models, *J. African Earth Sci.* 173 (2021) 104027, <https://doi.org/10.1016/j.jafrearsci.2020.104027>.
- [10] S. Erol, B. Erol, A comparative assessment of different interpolation algorithms for prediction of GNSS/levelling geoid surface using scattered control data, *Meas. J. Int. Meas. Confed.* 173 (2021) 108623, <https://doi.org/10.1016/j.measurement.2020.108623>.
- [11] M. McNut, Gravity, *Encycl. Ocean Sci.* (2019) 20–27. <http://doi.org/10.1016/B978-0-12-409548-9.11225-4>.
- [12] C. Braitenberg, The Global Gravity Field in Spherical Harmonics and Terrestrial Gravity Data, in: S. Elias, D. Alderton (Eds.), *Encycl. Geol.*, 2nd Ed., Academic Press, Oxford, 2020: pp. 706–718. doi:10.1016/B978-0-08-102908-4.00182-X.
- [13] Michel Van Camp, Olivier de Viron, Arnaud Watlet, Bruno Meurers, Olivier Francis, Corentin Caudron, Geophysics From Terrestrial Time-Variable Gravity Measurements, *Rev. Geophys.* 55 (4) (2017) 938–992, <https://doi.org/10.1002/rog.v55.410.1002/2017RG000566>.
- [14] P. Křen, On the inconsistency in experimental results of the gravitational constant, *Meas. J. Int. Meas. Confed.* 136 (2019) 622–624, <https://doi.org/10.1016/j.measurement.2019.01.007>.
- [15] T. Miwa, R. Hisakata, H. Kaneko, Effects of the gravity direction in the environment and the visual polarity and body direction on the perception of object motion, *Vision Res.* 164 (2019) 12–23, <https://doi.org/10.1016/j.visres.2019.08.005>.
- [16] P. Křen, V. Pálinská, M. Vaško, P. Mašika, Improved measurement model for FG5/X gravimeters, *Meas. J. Int. Meas. Confed.* 171 (2021) 108739, <https://doi.org/10.1016/j.measurement.2020.108739>.
- [17] Zhang Chijun, Bian Shaofeng, Yi Zhourun, Liu Lingtao, Fang Jian, Refining geoid and vertical gradient of gravity anomaly, *Geod. Geodyn.* 2 (4) (2011) 1–9, <https://doi.org/10.3724/SP.J.1246.2011.00009>.
- [18] M. Šprlák, S.-C. Han, W.E. Featherstone, Crustal density and global gravitational field estimation of the Moon from GRAIL and LOLA satellite data, *Planet. Space Sci.* 192 (2020) 105032, <https://doi.org/10.1016/j.pss.2020.105032>.
- [19] A.P. Yefremov, A.A. Vorobyeva, A planet's gravity assist as a powerful amplifier of small physical effects in the Solar system, *Acta Astronaut.* 180 (2021) 205–210, <https://doi.org/10.1016/j.actaastro.2020.12.029>.
- [20] RENISHAW Encoder product configurator, (n.d.). <https://www.renishaw.com/epc/en/encoder-product-configurator-6474>.
- [21] Ansys, (n.d.). <https://www.ansys.com/>.
- [22] Vatican Observatory, (n.d.). <http://www.vaticanobservatory.va/content/specolavaticana/en.html>.
- [23] Astro-Physics Command Center, (n.d.). https://www.siriussimaging.com/Help/APCC/advanced_pointing_model.htm.